Resonant Behavior of a Symmetric Missile Having Roll Orientation-Dependent Aerodynamics

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The transient response of a symmetric cruciform missle having roll orientation-dependent aerodynamics is studied. Aerodynamic force and moment expansions, cubic in angle of attack and harmonic in missle roll orientation, are incorporated into the equations of angular motion. An approximate resonant solution, generated by the method of averaging, is used to identify the special conditions of zero spin, fundamental resonance, and approximately one-half fundamental resonant spin. The roll orientation-dependent restoring and side moments are shown to have a dramatic influence on missile stability at the fundamental resonance roll rate. This work identifies the inherent tetragonal symmetry of the missile as a potential source of resonant behavior, in contrast to the established mechanism of configurational asymmetries.

Nomenclature

C_D	= missile drag coefficient
C_M	= in-plane static moment coefficient
$(C_{\tilde{m}}+iC_{\tilde{n}})$	= complex transverse aerodynamic moment coefficient expressed in the nonrolling frame
$C_{m_{\alpha_0}}$	= unmodulated restoring moment coefficient derivative
$C_{m_q}+C_{m_{\dot\alpha}}$	= pitch damping coefficient derivative
$C_{m_{\gamma\delta^2}}, C_{m_{\gamma\delta^4}}$	= coefficients of the roll orientation-dependent static moment expansion
$C_{m_{p\alpha}}$	= linear magnus moment coefficient derivative
C_N	= in-plane normal force coefficient
$C_{N_{\alpha_0}}$	= unmodulated normal force coefficient derivative
$C_{N_{\gamma\delta^2}}, C_{N_{\gamma\delta^4}}$	= coefficients of the roll orientation-dependent normal force expansion
C_{SM}	= side moment coefficient
C_{SN}	= side force coefficient
$(C_{\tilde{y}}+iC_{\tilde{z}})$	= complex transverse force coefficient expressed in the nonrolling frame
d	= missile reference diameter, cm
$(G_{\tilde{y}}+iG_{\tilde{z}})$	= transverse components of gravity in the nonrolling frame
I_x, I_y	= missile polar and transverse moments of inertia, kgm-m^2
K_1, K_2	= nutation and precession modal amplitudes
m	= missile mass, kgm
p	= roll rate, rad/s
$\bar{p}$	= pd/V
$(p,\tilde{q},\tilde{r})$	= missile angular velocity expressed in the nonrolling frame
$\hat{R}$	= $R/(\omega_1-\omega_2)$
S	= missile reference area, cm^2
s	= nondimensional arc length along the trajectory
$(u,\tilde{v},\tilde{w})$	= missile velocity vector expressed in the nonrolling frame
V	= magnitude of the missle velocity, m/s
(x,y,z)	= inertial frame coordinates
(X,Y,Z)	= missile-fixed coordinate frame
$(X,\tilde{Y},\tilde{Z})$	= missle nonrolling coordinate frame
γ	= $\theta-\hat{\phi}$, missile roll orientation relative to the plane of crossflow
δ	= magnitude of the complex angle of attack, $\tilde{\xi}$
θ	= orientation of the complex angle of attack in the nonrolling frame
θ_1,θ_2	= nutation and precession mode slowly varying phase
$\tilde{\mu}$	= $(\tilde{q}+i\tilde{r})d/V$, complex transverse angular velocity expressed in the nonrolling frame
$\tilde{\xi}$	= $(\tilde{v}+i\tilde{w})/V$, complex angle of attack expressed in the nonrolling frame
ρ	= air density, kgm/m^3
$\hat{\phi}$	= missle roll orientation
ψ_j	= total phase of the jth modal amplitude, K_j
ω_j	= frequency of jth modal amplitude, K_j
Subscripts	
0	= initial condition
$\overline{(\ )}$	= denotes complex conjugate
$(\ )^*$	= denotes multiplication by the density factor $(\rho Sd/2m)$

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Introduction

OVER the past several years, there has been a considerable amount of work done in an effort to explain the sometimes anomalous behavior of a free-flight missile in the vicinity of resonance. This resonant condition arises as the missile rolling velocity becomes sufficiently close to the missile's natural frequency (fundamental resonance), causing a corresponding amplification of the nonrolling trim angle of attack.

The transient response of a slightly asymmetric missile, spinning in the vicinity of its fundamental resonant frequency, has been investigated by several authors,[1-4] including the case of roll rate varying slowly through fundamental resonance.[5] More recently, Murphy[6] has shown the existence of missile subharmonic response due to nonlinearities in the aerodynamic force and moment system. This more general treatment of missile resonant response, rather than a restriction to fundamental resonance as predicted by the linear theory, has been a major contribution to an understanding of missile resonant behavior. While these works have thoroughly explored the possibilities of missile resonant behavior, they have had one basic assumption in common; i.e., the existence of a small, configurational asymmetry, which is the cause of the subsequent resonant behavior.

If the resonance phenomenon is considered from a more general viewpoint, it seems plausible that any aerodynamic force or moment which is periodic in missile roll orientation offers the potential of exciting missile resonant behavior. This resonant behavior, arising from the periodic variation of missile aerodynamics with roll position, is precisely the subject of this paper.

Roll Orientation-Dependent Aerodynamics

For the case of a symmetric, cruciform missile, the in-plane static restoring moment is observed to be not only a function of angle of attack, but is also dependent on the orientation of the plane of the angle of attack relative to the cruciform, as shown in Fig. 1. Experimentally, this variation of the static restoring moment with roll orientation is often observed to be appreciable.[7-9] Typical variations of the in-plane force and moment coefficients with roll orientation are shown in Fig. 2.

In addition to static restoring moment variations with roll orientation, experiment has also shown the existence of a roll orientation-dependent side moment and its corresponding side force. This "induced" side moment is caused by the lack of complete axial symmetry introduced by the presence of the fins. Thus, when the cross flow is seen to impinge on an asymmetric body cross section at specific roll orientation angles, a side-moment, arising from this asymmetric pressure distribution, results. Wind tunnel tests of typical missile configurations[7-9] have shown these side forces and moments to be reasonably described by harmonic functions of the roll orientation angle γ (Fig. 3).

Given some basic assumptions concerning the origin of the fluid forces acting on a cruciform missile, Maple and Synge[10] have shown that simple rotational and reflectional symmetry considerations often place unexpected restrictions on the functional form of the missile aerodynamic force and moment expansion. The essence of this method is embodied in the expansion of the fluid forces and moments in powers of cross-translational and angular velocity components, and a consideration of these expansions under coordinate transformation.

The Maple-Synge expansion may be written in the nonrolling or "aeroballistic" coordinate system $(X,\tilde{Y},\tilde{Z})$, which is allowed to pitch and yaw with the missile, but does not roll as illustrated in Fig. 1. If attention is restricted to static, roll orientation-dependent terms harmonic in 4γ, the following transverse force and moment coefficient expansions may be obtained:

$$(C_{\tilde{y}}+iC_{\tilde{z}}) = [-C_{N_{\alpha_0}}+C_{N_{\gamma\delta^2}}\delta^2 e^{-i4\gamma}+C_{N_{\gamma\delta^4}}\delta^4 e^{i4\gamma}+\cdots]\tilde{\xi} \quad (1)$$

and

$$(C_{\tilde{m}}+iC_{\tilde{n}}) = i[-C_{m_{\alpha_0}}+C_{m_{\gamma\delta^2}}\delta^2 e^{-i4\gamma}+C_{m_{\gamma\delta^4}}\delta^4 e^{i4\gamma}+\cdots]\tilde{\xi} \quad (2)$$

Inspection of Eqs. (1) and (2) reveals that the effects of roll orientation must be limited to terms cubic in angle of attack and higher. Furthermore, if the expansion is truncated at terms cubic in angle of attack, it is evident that the magnitude of the in-plane and out-of-plane contributions due to roll orientation are equal.

If nonlinearities in the normal force and restoring moment are deleted from consideration, and the usual damping and Magnus moments are included, the roll orientation-dependent force and moment expansions become

$$(C_{\tilde{y}}+iC_{\tilde{z}}) = [-C_{N_{\alpha_0}}+C_{N_{\gamma\delta^2}}\delta^2 e^{-i4\gamma}]\tilde{\xi} \quad (3)$$

$$(C_{\tilde{m}}+iC_{\tilde{n}}) = [-iC_{m_{\alpha_0}}+iC_{m_{\gamma\delta^2}}\delta^2 e^{-i4\gamma} + C_{m_{p\alpha}}\bar{p}]\tilde{\xi} - iC_{m_{\dot{\alpha}}}\tilde{\xi}' + C_{m_q}\tilde{\mu} \quad (4)$$

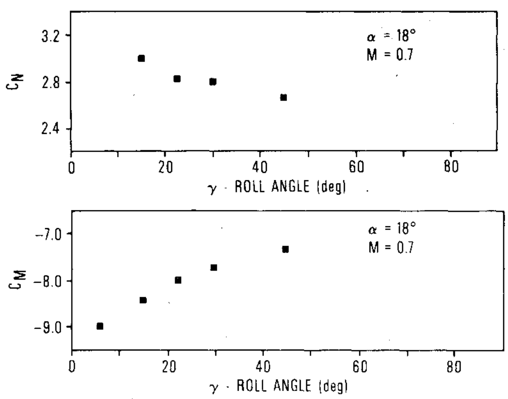

Fig. 2 Experimental roll orientation-dependent in-plane force and moment coefficient data.[7,8]

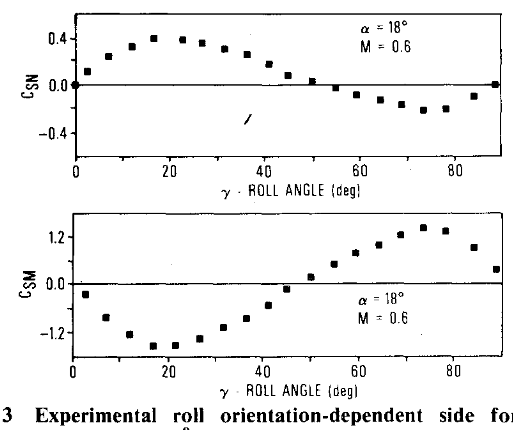

Fig. 3 Experimental roll orientation-dependent side force and moment coefficient data.[9]

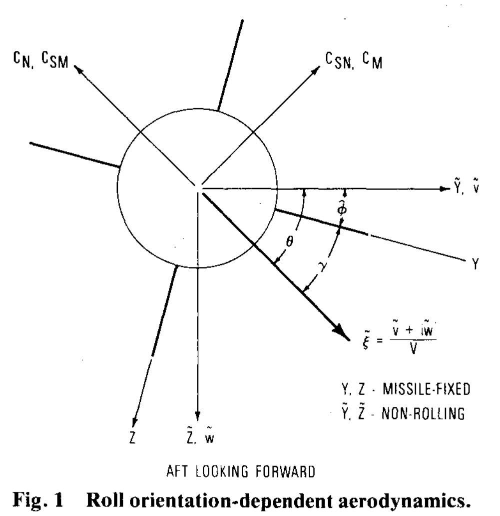

Fig. 1 Roll orientation-dependent aerodynamics.

where the roll orientation-dependent terms have been limited to third order in δ.

Equations of Motion and the Method of Averaging

The equations of angular motion of a rigid, inertially symmetric missile may be written in terms of the complex angle of attack $\tilde{\xi}$ and the complex angular velocity $\tilde{\mu}$ expressed in the nonrolling frame. Many excellent developments of the equations of motion may be found in the literature,[2,11] and for the sake of brevity, will not be repeated here. Instead, the nomenclature and coordinate frames of Ref. 2 will be adopted to express the complex force and moment differential equations as

$$\tilde{\xi}' - C_D^*\tilde{\xi} - i\tilde{\gamma}\tilde{\mu} = (C_{\tilde{y}}^* + iC_{\tilde{z}}^*) + (G_{\tilde{y}} + iG_{\tilde{z}})d/V^2 \tag{5}$$

$$\tilde{\mu}' - C_D^*\tilde{\mu} - iP\tilde{\mu} = (C_{\tilde{m}}^* + iC_{\tilde{n}}^*)k_t^{-2} \tag{6}$$

where

$$\tilde{\gamma} = u/V \tag{7}$$

$$P = (pd/V)(I_x/I_y) \tag{8}$$

$$k_t^{-2} = md^2/I_y \tag{9}$$

and $(\ )'$ denotes differentiation with respect to nondimensional arc length, s.

With the assumption of constant flight conditions, a restriction to small geometrical angles and the omission of gravity terms, Eqs. (3) and (4) may be combined with Eqs. (5) and (6) to obtain a nonlinear differential equation in the complex angle of attack:

$$\tilde{\xi}'' + (H-iP)\tilde{\xi}' - (M+iPT)\tilde{\xi} = (R+iPS_\gamma)\delta^2 e^{-i4\gamma}\bar{\tilde{\xi}} + 3C^*_{N_{\gamma\delta^2}}\delta^2 e^{-i4\gamma}(\bar{\tilde{\xi}})'\tilde{\xi}/\bar{\tilde{\xi}} \tag{10}$$

where

$$H = C_{N_{\alpha_0}} - (C^*_{m_q} + C^*_{m_{\dot{\alpha}}})k_t^{-2} \tag{11}$$

$$M = k_t^{-2}C^*_{m_{\alpha_0}} \tag{12}$$

$$T = C^*_{N_{\alpha_0}} + k_a^{-2}C^*_{m_{p\alpha}} \tag{13}$$

$$R = -k_t^{-2}C^*_{m_{\gamma\delta^2}} \tag{14}$$

$$S_\gamma = C^*_{N_{\gamma\delta^2}}[(4I_y/I_x) - 1] \tag{15}$$

and

$$k_a^{-2} = md^2/I_x \tag{16}$$

Clearly, the effects of the roll orientation-dependent aerodynamics are manifest by the nonhomogeneous terms of Eq. (10).

Consideration of Eq. (10) as the aerodynamic damping, Magnus, and roll orientation-dependent terms become vanishing small yields

$$\tilde{\xi}'' - iP\tilde{\xi}' - M\tilde{\xi} = 0 \tag{17}$$

Solutions to this unperturbed, gyroscopic, linear differential equation are of the form

$$\tilde{\xi} = \sum_{j=1}^{2} K_j e^{i\psi_j} \tag{18}$$

and

$$\tilde{\xi}' = \sum_{j=1}^{2} iK_j\omega_j e^{i\psi_j} \tag{19}$$

where

$$\psi_j = \omega_j s + \theta_j \tag{20}$$

and K_j and θ_j are real constants depending on initial conditions.

In the case of the complete, nonlinear Eq. (10), it is assumed that both $\tilde{\xi}$ and $\tilde{\xi}'$ are given by solutions of Eq. (17). However, in a manner similar to that of Ref. 12, allowance is made for the variation of the amplitude and phase of each mode, while simultaneously imposing Eqs. (18) and (19). This condition is shown to be equivalent to the condition

$$\sum_{j=1}^{2}(K_j' + iK_j\theta_j')e^{i\psi_j} = 0 \tag{21}$$

Substitution of Eqs. (18) and (19) into Eq. (10) while incorporating Eq. (21) yields the approximate amplitude and phase differential equations

$$iK_1'(\omega_1-\omega_2) + \theta_1'(\omega_2-\omega_1)K_1 + i(H\omega_1 - PT)K_1 = \alpha_1 - iK_2(H\omega_2 - PT)e^{i(\psi_2-\psi_1)} \tag{22}$$

$$iK_2'(\omega_2-\omega_1) - \theta_2'(\omega_2-\omega_1)K_2 + i(H\omega_2 - PT)K_2 = \alpha_2 - iK_1(H\omega_1 - PT)e^{i(\psi_1-\psi_2)} \tag{23}$$

where

$$\alpha_j = F(\gamma,\delta,\theta)\sum_{\ell=1}^{2}K_\ell e^{i(\psi_\ell - \psi_j)} \tag{24}$$

and

$$F(\gamma,\delta,\theta) = [(R+iPS_\gamma) + 3C^*_{N_{\gamma\delta^2}}(\bar{\tilde{\xi}})'/\bar{\tilde{\xi}}]\delta^2 e^{-i4\gamma} \tag{25}$$

Examination of Eqs. (22) and (23) reveals that the dominant components of the α_j's contain terms of frequency $4(\bar{p}-\omega_1)$, $(4\bar{p}-3\omega_1-\omega_2)$, $(4\bar{p}-2\omega_1-2\omega_2)$, $(4\bar{p}-\omega_1-3\omega_2)$, and $4(\bar{p}-\omega_2)$. Therefore, an approximate solution to Eq. (10) is sought by averaging Eqs. (22) and (23) over an arbitrary interval which is large with respect to the wavelength of the lowest frequency term of the α_j's.

$$\{iK_1'(\omega_1-\omega_2) + \theta_1'(\omega_2-\omega_1)K_1 + i(H\omega_1-PT)K_1\}_{\text{ave}} = I_1 \tag{26}$$

$$\{iK_2'(\omega_2-\omega_1) - \theta_2'(\omega_2-\omega_1)K_2 + i(H\omega_2-PT)\}_{\text{ave}} = I_2 \tag{27}$$

where

$$I_j = \lim_{L\to\infty}\frac{1}{2\pi L}\int_{\hat{\psi}_0}^{\hat{\psi}_0+2\pi L}\alpha_j d\hat{\psi} \tag{28}$$

and

$$\hat{\psi} = \psi_1 - \psi_2 \tag{29}$$

Performing the averaging process as indicated by Eq. (28), it may be shown that the α_j's have nonzero average values for only three physically possible values of spin. These values are zero spin, fundamental resonance ($\bar{p}=\omega_1$), and approximately one-half fundamental resonant spin, $\bar{p}=(3\omega_1+\omega_2)/4\approx\omega_1/2$. The remainder of this paper is devoted to an investigation of the missile behavior at these three special spin rates.

Zero Spin

For the case of zero spin, the α_j's may be averaged with $\bar{p}=0$ to obtain the following contributions to the approximate amplitude and phase differential equations:

$$I_1 = 3K_1K_2^2 e^{i\beta_1}\,[R - iC^*_{N_{\gamma\delta^2}}\,(\omega_1 + 2\omega_2)] \tag{30}$$

$$I_2 = 3K_1^2K_2 e^{i\beta_1}\,[R - iC^*_{N_{\gamma\delta^2}}\,(2\omega_1 + \omega_2)] \tag{31}$$

where

$$\beta_1 = 4[\hat{\phi}_0 - (\theta_1 + \theta_2)/2] \tag{32}$$

Inspection of Eqs. (30) and (31) for typical missile aerodynamic and inertial properties reveals that the terms within the brackets may be approximated by the real part only. Hence, for the case of zero spin, Eqs. (26) and (27) may be approximated by

$$K_1' = -1/2\ HK_1 + 3\hat{R}K_1K_2^2\ \sin\beta_1 \tag{33}$$

$$K_2' = 1/2\ HK_2 - 3\hat{R}K_1^2K_2\ \sin\beta_1 \tag{34}$$

$$\theta_1' = -3\hat{R}K_2^2\ \cos\beta_1 \tag{35}$$

$$\theta_2' = 3\hat{R}K_1^2\ \cos\beta_1 \tag{36}$$

The apparent symmetry of Eqs. (33) and (34) in the amplitudes K_1 and K_2 may be exploited by defining the mean squared amplitude δ_M^2 as

$$\delta_M^2 \equiv K_1^2 + K_2^2 \tag{37}$$

Eqs. (33) and (34) may be added after multiplication by K_1 and K_2, respectively, while incorporating Eq. (37) to obtain

$$(\delta_M^2)' = -H\delta_M^2 \tag{38}$$

Thus, Eq. (38) suggests that the roll orientation-dependent restoring and side moments, while having a possible influence on the phase of the motion, have little effect on the stability of the modal amplitudes at zero spin.

The validity of these conclusions, for the case of zero spin, was verified by a comparison of numerical solutions of Eqs. (33-36) with direct solutions of Eq. (10). Missile configurational properties, flight conditions, and aerodynamics used in the calculations may be found in Table 1. In Fig. 4, $C_{N_{\gamma\delta^2}}$ and $C_{m_{\gamma\delta^2}}$ have been included in the aerodynamic force and moment expansion and the resulting numerical solution of Eq. (10) compared with a solution in which the roll orientation-dependent terms were omitted. The δ^2 history is seen to be slightly altered both in damping and phase. However, the δ_M^2 histories, as predicted by Eq. (38), are virtually identical.

Table 1 Missile configurational properties

Mach number	0.89
Reference diam, d	27.3 cm
Mass, m	272.1 kgm
Polar moment of inertia, I_x	2.72 kgm-m^2
Transverse moment of inertia, I_y	54.5 kgm-m^2
Aerodynamics	
$C_{n_{\alpha_0}} = 4.3$	$C_{m_q} + C_{m_{\dot\alpha}} = -50.0$
$C_{N_{\gamma\delta^2}} = 9.0$	$C_{m_{p_\alpha}} = 5.0$
$C_{m_{\alpha_0}} = -4.2$	$C_{m_{\gamma\delta^2}} = -60.0$

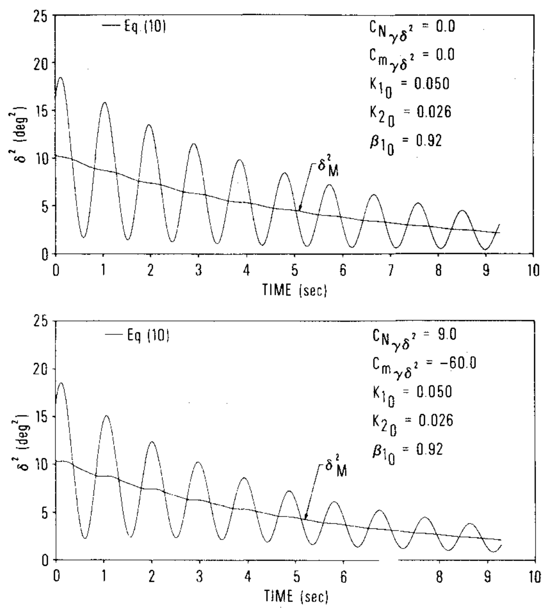

Fig. 4 Missile response to initial conditions at zero spin.

Fundamental Resonance

The approximate amplitude and phase differential equations for the case of fundamental resonance spin may be obtained through the use of Eq. (28) with $\bar{p} = \omega_1$. The resulting nonlinear system describing the variation of the K_j's and θ_j's is found to be:

$$K_1' = \lambda_1 K_1 + K_1^3\,[\hat{R}\ \sin\beta_2 + \omega_1 \hat{C}^*_{N_{\gamma\delta^2}}\ \cos\beta_2] \tag{39}$$

$$K_2' = \lambda_2 K_2 \tag{40}$$

$$\theta_1' = -K_1^2\,[\hat{R}\ \cos\beta_2 - \omega_1 \hat{C}^*_{N_{\gamma\delta^2}}\ \sin\beta_2] \tag{41}$$

$$\theta_2' = 0 \tag{42}$$

where

$$\lambda_1 = -(H\omega_1 - PT)/(\omega_1 - \omega_2) \tag{43}$$

$$\lambda_2 = (H\omega_1 - PT)/(\omega_1 - \omega_2) \tag{44}$$

$$\beta_2 = 4(\hat{\phi}_0 - \theta_1) \tag{45}$$

$$\hat{C}^*_{N_{\gamma\delta^2}} = C^*_{N_{\gamma\delta^2}}/(\omega_1 - \omega_2) \tag{46}$$

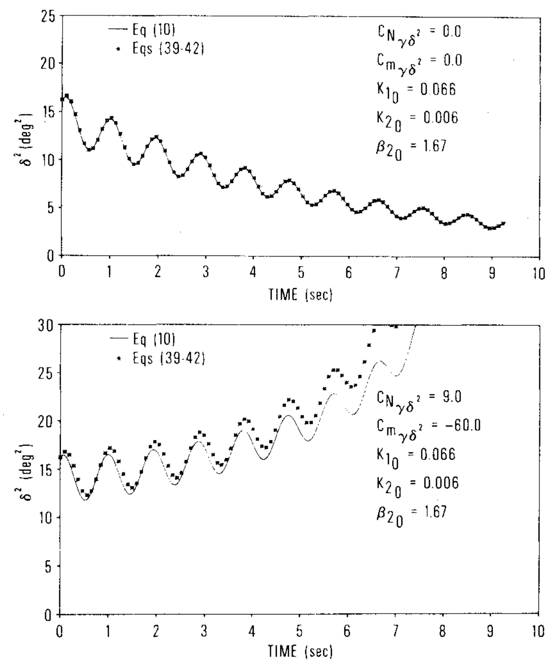

Fig. 5 Missile response to initial conditions at fundamental resonance.

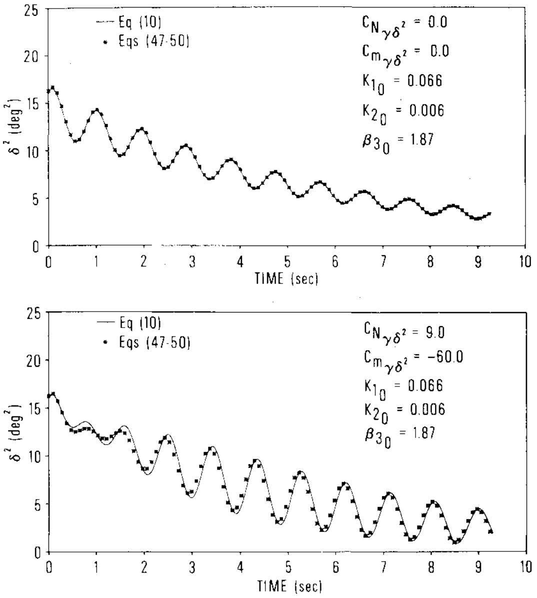

Fig. 6 Missile response to initial conditions at one-half fundamental resonant spin.

and the moment of inertia ratio, I_x/I_y, has been neglected in comparison with unity. As indicated by Eqs. (39-42), the effects of the roll orientation-dependent aerodynamics to a first approximation are confined to the 1 mode (nutation).

Numerical solutions of Eqs. (39-42) are compared with direct solutions of Eq. (10) in Fig. 5. The inclusion of $C_{N_{\gamma\delta^2}}$ and $C_{m_{\gamma\delta^2}}$ in the force and moment expansion is sufficient to cause the missile, with a supposed respectable level of aerodynamic damping, to exhibit divergent angular motion while at fundamental resonant spin. Through the use of a phase plane analysis of Eqs. (39-42), the stability of motion is seen to depend on initial conditions defined by K_l^2 and β_2, as well as the relative magnitudes of λ_l and $\hat{R}$.

One-Half Resonant Spin

The final spin rate which results in nonzero average values for the α_j's is the case $\bar{p}=(3\omega_l+\omega_2)/4$ or $\bar{p}\approx\omega_l/2$. The appropriate damping and phase differential equations, as derived from Eq. (28), are:

$$K_l' = \lambda_l K_l + 3K_l^2 K_2 [\hat{R}\sin\beta_3 + \omega_l \hat{C}^*_{N_{\gamma\delta^2}}\cos\beta_3] \tag{47}$$

$$K_2' = \lambda_2 K_2 - K_l^3 [\hat{R}\sin\beta_3 + \omega_2 \hat{C}^*_{N_{\gamma\delta^2}}\cos\beta_3] \tag{48}$$

$$\theta_l' = -3K_l K_2 [\hat{R}\cos\beta_3 - \omega_l \hat{C}^*_{N_{\gamma\delta^2}}\sin\beta_3] \tag{49}$$

$$\theta_2' = (K_l^3/K_2)[\hat{R}\cos\beta_3 - \omega_2 \hat{C}^*_{N_{\gamma\delta^2}}\sin\beta_3] \tag{50}$$

where

$$\beta_3 = 4\hat{\phi}_0 - 3\theta_l - \theta_2 \tag{51}$$

In contrast to the case of fundamental resonant spin, both the 1 and 2 modes are affected by the presence of the roll orientation-dependent aerodynamics at one-half the resonant roll rate.

The effect of the roll orientation-dependent aerodynamics on missile response to initial conditions at one-half fundamental resonant spin is depicted by the graphs of Fig. 6. For the initial conditions investigated, it appears that the inclusion of $C_{N_{\gamma\delta^2}}$ and $C_{m_{\gamma\delta^2}}$ results in a slight alteration of the missile angular motion history, as shown in Fig. 6. However, Eqs. (47) and (48) suggest that this effect is more pronounced in regions where the missile linear damping rates (λ_j's) are marginal. A complete analysis of the nonlinear system composed of Eqs. (47-50) is necessary before any conclusion regarding stability or the possible existence of stable limit motions can be made.

Conclusions

The angular motion of a symmetric, cruciform missile in the presence of nonlinear roll orientation-dependent aerodynamics has been investigated. The method of averaging was employed to obtain approximate solutions to the equations of angular motion, and to identify those spin rates at which the roll orientation-dependent aerodynamics might play a significant roll in determining missile stability.

For the case of zero spin, the roll orientation-dependent restoring and side moments were found to have little effect on the missile angular motion. The behavior of the modal amplitudes, to a first approximation, was found to be virtually unaffected by the roll orientation-dependent aerodynamics. The most pronounced influence of the roll orientation-dependent static moment was found to be at fundamental resonant spin, where the inclusion of roll dependence was sufficient to cause missile divergent behavior. The condition of one-half fundamental resonant spin was also seen to exhibit a sensitivity to roll orientation-dependent aerodynamics for the conditions investigated. However, further analysis and calculations are necessary before conclusions regarding the stability of the motion can be made.

Acknowledgment

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